* M C R - 0 0 3 - 0 0 1 5 1 5 - 1 2 0 - 4 *

MCR-003-001515 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

May / June – 2018

Mathematics : 503 (A)

(Discrete Mathematics & Complex Analysis - I)

(New Course)

Faculty Code : 003

Subject Code : 001515

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

(1) All questions are compulsory.

(2) Figures to the right side indicate marks.

1 Answer all the following 20 short questions : **20**

(1) Define : Antisymmetric Relation.

(2) Define : Bounded Lattice.

(3) For the Poset (S_{30}, D), $3' =$ _____.

(4) Define : Partial order Relation.

(5) Define : Lattice Isomorphism.

(6) Define : Boolean Algebra.

(7) Find the atoms of Boolean Algebra $(S_{30}, *, \oplus, ', 0, 1)$.

(8) Define : Atoms in Boolean Algebra.

(9) The sum of all minterms of n-variables is _______.

(10) If $(L, *, \oplus, 0, 1)$ is bounded lattice then $a \oplus 0 =$ ______.

(11) Write Laplace equation in polar form.

(12) Define : Harmonic function.

(13) Write the value of $\frac{dw}{dz}$ in polar form.

(14) Determine $\exp(z)$ is either analytic function or not ?

(15) Imaginary part of $\frac{2+3i}{3-4i}$ is ______.

(16) If $c : |z-2| = \frac{1}{2}$ then $\int_c \frac{dz}{z-3}$ = ______.

(17) If $c : |z| = 3$ then $\int_c \frac{z^2}{z-3}\, dz$ = ______.

(18) State fundamental theorem of Algebra.

(19) Evaluate : $\int_c \frac{z+2}{z}\, dz$ where c is the circle $z = 2e^{i\theta}$, where $0 \le \theta \le 2\pi$.

(20) Find the value of $\int_0^{2\pi} \cos\left(\frac{z}{2}\right) dz$ in the exponential form.

2 (a) Attempt any three : **6**

(1) Consider the Relation $R = \{(i, j) \mid |i - j| = 2\}$ on $\{1, 2, 3, 4, 5, 6\}$. Is R transitive ?

(2) Define : Meet and Join.

(3) In a complemented distributive lattice show that $a \wedge b' = 0 \Rightarrow a' \vee b = 1$.

(4) $(B, *, \oplus, ', 0, 1)$ is Boolean Algebra then prove that $(a')' = a$ where $a \in B$.

(5) Let $(B, *, \oplus, ', 0, 1)$ is Boolean Algebra then $\forall\, a, b \in B$ prove that $a \le b \Rightarrow a * b' = 0$.

(6) If a and b are distinct atoms of the Boolean Algebra $(B, *, \oplus, ', 0, 1)$ then prove that $a * b = 0$.

(b) Attempt any three : **9**

(1) Z be the set of integers and given $R = \{(x, y) \mid x - y$ is divisible by 5$\}$ check whether R is an equivalence Relation or not.

(2) In usual notation show that (S_6, D) is a lattice.

(3) Give an example of a bounded lattice which is not complemented lattice.

(4) Prove that A non zero element a of Boolean Algebra $(B, *, \oplus, ', 0, 1)$ is an atom iff $\forall\ x \in B$ either $a * x = 0$ or $a * x = a$.

(5) If $(B, *, \oplus, ', 0, 1)$ is Boolean Algebra then prove that for any $x_1, x_2 \in B,\ A\left(x_1 * x_2\right) = A\left(x_1\right) \cap A\left(x_2\right)$.

(6) Express $\alpha\left(x_1, x_2, x_3\right) = x_1 + x_2$ as "Sum of product canonical form".

(c) Attempt any two : **10**

(1) State and prove Distributive inequality.

(2) Prove that every chain is a distributive lattice.

(3) Let L_1 be the lattice $D_6 = \{1, 2, 3, 6\}$ (divisor of 6) and L_2 be the lattice $(P(s), \subseteq)$ where $S = \{a, b\}$. Then show that D_6 is Isomorphic to P(s).

(4) State and prove D' Morgan's law for the Boolean Algebra.

(5) Write all the minterms of the two and three variables.

3 (a) Attempt any three : **6**

(1) Show that $f(z) = z - \bar{z}$ is not an analytic function.

(2) Define : Limit of a complex function.

(3) If $f = u + i\vartheta$ and its complex conjugate $\bar{f} = u - i\vartheta$ are analytic then show that f is constant.

(4) Evaluate $\int_{i}^{i/2} e^{\pi z}\, dz$.

(5) Evaluate $\int_c \frac{z\, dz}{\left(q - z^2\right)(z + i)}$ where c be the positively oriented circle $|z| = 2$.

(6) State Cauchy inequality.

(b) Attempt any three : **9**

(1) If u and v are conjugate harmonic function then prove that the family of curves obtained by $u = c_1$ and $v = c_2$ are orthogonal.

(2) Prove that $u = r^2 \sin 2\theta$ is a harmonic function and find its conjugate.

(3) Find an analytic function $f(z)$ whose real part is $\cos x \cdot \cosh y$.

(4) Find the value : $\int_c \frac{dz}{z^2+4}$, $c : |z-i| = 2$.

(5) State and prove Cauchy's fundamental theorem.

(6) Prove that $\left| \int_c \frac{z+4}{z^3-1} dz \right| \leq \frac{6\pi}{7}$ where c be the arc of the circle $|z| = 2$ from $z = 2$ to $z = 2i$.

(c) Attempt any two : **10**

(1) Obtain Cauchy-Riemann condition for an analytic function $f(z)$ is polar form.

(2) Prove that the analytic function of constant modulus is also constant in its domain D.

(3) Prove that $f(z) = \begin{cases} \frac{(\bar{z})^2}{z} & , \; z \neq 0 \\ 0 & , \; z = 0 \end{cases}$

satisfied Cauchy-Riemann conditions at origin however $f(z)$ is not analytic function at origin.

(4) Find the value of $\int_c (3z+1)\, dz$ where c is a square joining points $z = 0$, $z = 1$, $z = i$ and $z = 1 + i$.

(5) State and prove Liouville's theorem.